Thermo Derivation

Find R

$$H = U + PV$$

$$dH = dU + d(PV) \qquad dU = -PdV = nc_v dT$$

$$dH = -PdV + PdV + Vdp$$

$$dH = Vdp \qquad dH = nc_p dT$$

$$nC_p dT = Vdp$$

$$PdV + Vdp = nRdT$$

$$-nC_v dT + nC_p dT = nRdT \qquad (\text{div. } ndT)$$

$$-C_v + C_p = R$$

or $\underline{C_p - C_v = R}$

Find $PV^\gamma = C$

$$pdV + Vdp = nRdT \quad , \quad nC_v dT = -PdV$$

$$ndT = \frac{-PdV}{C_V}$$

$$pdV + Vdp = \frac{-pdV}{C_V} R \qquad (\times C_V), (\div R)$$

$$\frac{C_V PdV}{R} + \frac{C_V}{R} Vdp = -pdV$$

$$PdV + \frac{C_V PdV}{R} + \frac{C_V}{R} Vdp = 0$$

$$PdV\left(1 + \frac{C_V}{R}\right) + \frac{C_V}{R} Vdp = 0$$

$R = c_p - c_v \qquad (\div R)$

$1 = \frac{c_p}{R} - \frac{c_v}{R}$

or $\frac{c_v}{R} + 1 = \frac{c_p}{R} \qquad$ (sub back in)

$\frac{c_p}{R} p\,dV + \frac{c_v}{R} V\,dp = 0 \qquad (\text{div. } \frac{c_v}{R})$

$\frac{c_p}{\cancel{R}} \frac{\cancel{R}}{c_v} p\,dV + V\,dp = 0$

$\frac{c_p}{c_v} p\,dV + V\,dp = 0 \qquad (c_p/c_v = \gamma)$

$\gamma p\,dV + V\,dp = 0 \qquad$ (div pV)

$\gamma \frac{dV}{V} + \frac{dp}{p} = 0 \qquad$ integrate

$\gamma \ln V + \ln p = c$

$$\underline{\underline{pV^{\gamma} = c}}$$